BOOK *of* INSTRUCTIONS (T)

·

For the use of

THE P.I.C.
DIFFERENTIAL
SCALES

for

Trigonometrical
Computations

Price One Shilling and Threepence

TRIGONOMETRICAL DIFFERENTIAL SCALES

(Designed under Patent No. 411090)
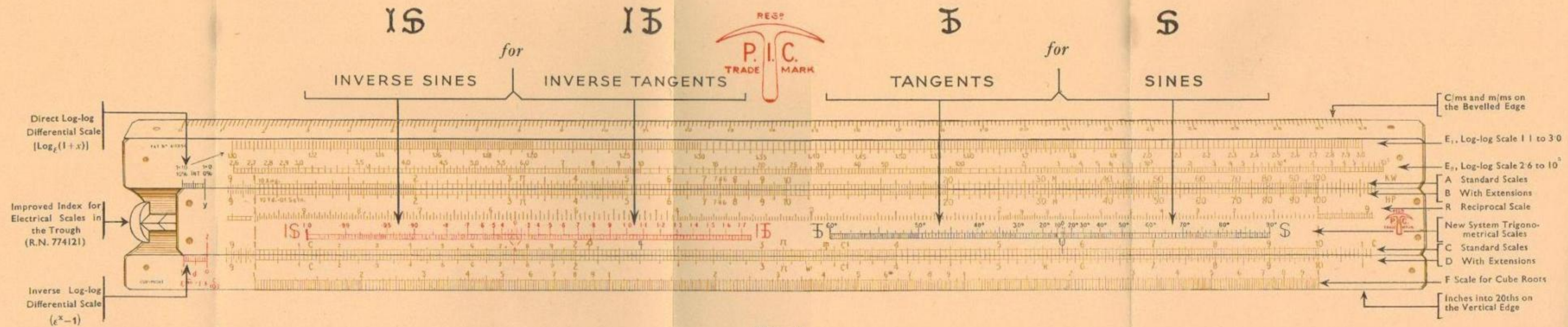
RES?
P. I. C.
TRADE MARK

for

INVERSE SINES **INVERSE TANGENTS** **TANGENTS** **SINES**

Direct Log-log Differential Scale [$\log_\varepsilon(1+x)$]

Improved Index for Electrical Scales in the Trough (R.N. 774121)

Inverse Log-log Differential Scale ($\varepsilon^x - 1$)

C/ms and m/ms on the Bevelled Edge

E_1, Log-log Scale 1·1 to 3·0

E_2, Log-log Scale 2·6 to 10^3

A Standard Scales
B With Extensions
R Reciprocal Scale

New System Trigonometrical Scales

C Standard Scales
D With Extensions

F Scale for Cube Roots

Inches into 20ths on the Vertical Edge

Fig. 1

SUPPLEMENT TO THE P.I.C.
BOOK *of* GENERAL
INSTRUCTIONS

Introducing the Use of

THE P.I.C.
DIFFERENTIAL
SCALES

for

Trigonometrical
Computations

by

H. Boardman

INSTRUCTIONS REGARDING
P.I.C.
TRIGONOMETRICAL DIFFERENTIAL SCALES

Those who are familiar with Slide Rule Art know that the standard practice in regard to Trigonometrical Calculations is to provide several lengths of logarithmic Sine and Tangent Scales. In order to effect economy of scale length, most orthodox designs sacrifice accuracy by relating the Sine scales to the "half length" A and B scales and only supply a "single scale length" to serve for both of the Sine and Tangent ranges 34 minutes to 5 degrees–45 minutes, an approximation which cannot be justified if the accuracy maintainable by the C and D scales is to be upheld. By this now obsolete method, for accuracy over a part range somewhat in accord with that possible with the C and D scales, four "full length" scales are required. Such scales or approximate equivalents cannot normally be accommodated on the face of the slide together with the B and C scales and have to be relegated to the under-side where they are not readily available for manipulation in conjunction with the other scales on the face of the Rule. For many, these scales serve only as reference conversion scales when trigonometrical tables are not to hand and in this respect for certain Inverse Sine determinations are little more than useless.

In contrast to reduction of scale length at the expense of accuracy by approximations or the relating of the trigono-metrical scales to the A and B scales, the P.I.C. new system of Direct and Inverse Sine and Tangent Differential Scales, which together comprise less than a "full scale length," are designed in relation to the D scale and give consistent maximum accuracy over an extended angle range. This efficient curtailment of scale length permits of these patented scales being accommo-dated on the slide between the C and D scales (as shown in

Fig. 1) where they are readily available for use in conjunction with the other scales on the face of the Rule. The advantages which accrue from this direct accessibility and increased angle range are obvious; experience with the scales will soon reveal their superiority as compared with the ordinary standard scales and furnish evidence that they are highly effective at all parts of the complete angle range—Direct and Inverse—and facilitate a degree of accuracy more consistently high.

In operation, the C and D scales of the designs which incorporate the Trigonometrical Differential Scales, in addition to serving for the ordinary number range, also function as Deci-Angle Scales. Angles (in degrees and decimals) are applied to the C or D scales and results that are measures of angles read from the D scale.

The Direct Sine and Tangent Differential Scales provide the necessary *divisor correctives* that must be applied to angle readings on C or D to provide the respective trigonometrical functions. Similarly, the Inverse Sine and Tangent Differential Scales provide the *divisor correctives* that must be applied to function values on D to furnish the corresponding angles on D at C1 or C10.

The appearance of, for example, the Sine Differential Scale in the region of the 0° to 30° mark, in the respect that the distance is so comparatively short and the divisions so few may suggest to the non-mathematical beginner that the accuracy is accordingly somewhat limited, but after a little practice with this scale, and thought regarding the nature of Sines, the user will correctly interpret the meaning of these small variations of the *divisor correctives* for the early angle range. Similar observations can be made with regard to the other Scales. After comparison with tabulated and calculated results, users will soon realise that the highest significant figure accuracy possible by the C and D scales is consistently maintained by the Differential Scales over the complete angle range.

On inspection of the Rule it will be observed that the Common Zero of the Direct Scales is at "U" and coincides with the C scale reading 57·3, *i.e.*, $\frac{180}{\pi}$. Inverse Scales have their Common Zero at "V" which corresponds to a C scale reading of 0·01746, *i.e.*, $\frac{\pi}{180}$. (The respective uses of the gauge marks U, *m*, and *s*, are enumerated on pages 16 and 17, where also the determination of Sines and Tangents of angles less than 3 degrees are specially considered).

Just as the letters A, B, C, and D are used in the general instructions in reference to the standard scales and K to the cursor line so will the following symbols be employed in regard to the Differential Scales:

S   refers to the Direct Sine Differential Scale

T   do.   do.   Direct Tangent   do.

IS   do.   do.   Inverse Sine   do.

IT   do.   do.   Inverse Tangent   do.

Thus **S**60° will refer to the 60° line of the Direct Sine Differential Scale and so on.

These "Differential Scales" are exceedingly simple to manipulate as will be seen from the following worked examples:
N.B.—It will be assumed that the user can manipulate the C and D scales and has a sufficient knowledge of Sines, tangents, etc., to enable the decimal points to be positioned in the various significant figure results obtained. The following table may prove useful at the outset.

x	0·001	0·01	0·1	1·0
$\mathrm{Sin}^{-1}x$	0°–3′–26″	0°–34′–23″	5°–44′–21″	90°
$\mathrm{Tan}^{-1}x$	0°–3′–26″	0°–34′–23″	5°–42′–38″	45°

x	10	100	1000
$\mathrm{Tan}^{-1}x$	84°–17′–22″	89°–25′–37″	89°–56′–34″

Example 1.

(a) *To determine the value of Sine 43°.*

Set the cursor at D43 and move the slide bringing S43° to the cursor. Read the value of Sine 43° on the D scale at (C1 or) C10. ... ... *Viz.* 0·6820.

(b) *To determine the value of Tan 36·5° (less than 60°).*

Set the cursor at D365 and move the slide bringing T36·5° to the cursor. Read the value of Tan 36·5° on the D scale at (C1 or) C10 ... ... *Viz.* 0·740

(c) *To determine the value of Tan 76° (greater than 60°)*

$$\text{Tan } 76° = \frac{1}{\text{Tan } (90° - 76°)} = \frac{1}{\text{Tan } 14°}$$

To D1 bring C14 and move the cursor to T14°. Read the value of Tan 74° on the D scale at the cursor.

Viz. 4·01

(d) *Express (i) in degrees (ii) in radians, the angle whose Tangent is 0·9, i.e., the value of Tan⁻¹ 0·9.*

Set the cursor at D9 and move the slide bringing T0·9 to the cursor. Read the degrees of angle on the D scale at C1 (or C10) ... ... *Viz.* 42°

For radians read on the D scale at " V." *Viz.* 0·733

(e) *Determine the value of the angle (in degrees and decimals) whose Sine is 0·66, i.e., the value of Sin⁻¹ 0·66.*

Set the cursor at D66 and move the slide bringing S0·66 to the cursor. Read the desired angle on the D scale at C1 (or C10). ... ... *Viz.* 41·3° or 41°—18'

It will be observed from parts (a), (b), (c), and (d) of the foregoing example that

$\text{Sin } 43°$ is treated as $\dfrac{43}{\text{S } 43°}$, $\text{Tan } 36·5°$ as $\dfrac{36·5}{\text{T } 36·5°}$,

$(\text{Tan}^{-1} 0·9)$ as $\dfrac{0·9}{\text{T } 0·9}$, $(\text{Sin}^{-1} 0·66)°$ as $\dfrac{0·66}{\text{S } 0·66}$.

(See Theoretical Considerations, pages 20 to 22)

Example 2.

(a) **To determine the value of 73 Sin 52°.**

Set K at D52, S52° to K, K to C73 and read the result on
D at K.   ...   .   *Viz.* 57·5

or alternatively

Set K at D73, S52° to K, K to C52 and read the result on
D at K., *i.e*, treat as $\dfrac{52 \times 73}{\text{S}52°}$ or $\dfrac{73 \times 52}{\text{S}52°}$

(b) **Similarly treat 26 Tan 41° as:-**

$$\dfrac{26 \times 41}{\text{T}41°} \quad \text{or} \quad \dfrac{41 \times 26}{\text{T}41°}$$

Set K at D 26, T41° to K, K to C41 and read the value on
D at K.   ...   .   *Viz.* 22·6

or alternatively

Set K at D41, T41° to K, K to C26 and read the result on
D at K   ...   .   *Viz.* 22·6

Example 3.

(a) **To determine the value of** $\dfrac{133}{\text{Sin }38°}$

treat as $\dfrac{133\,\text{S}38}{38}$

Set K at D133, C38 to K, K to S38° and read the result on
D at K   ...   .   *Viz.* 216·0

(b) **To determine the value of** $\dfrac{162}{\text{Tan }27°}$

treat as $\dfrac{162\,\text{T}27}{27}$

Set K at D162, C27 to K, K to T27° and read the result
on D at K...   ...   .   *Viz.* 318·0

In practice it will be found that the position of the
Trigonometrical Differential Scales on the slide between the
B and C scales enables most of the common evaluations to be
effected with absolute continuity of operation

The following typical cases regarding Solution of Triangles etc., are intended to serve as useful practice exercises:

Right-angled Triangles

In the accompanying reference right-angled triangle "a" is
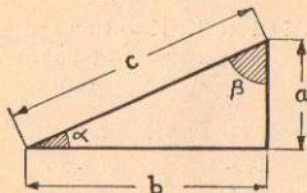
the shortest side subtending the smallest angle "α", (the magnitude of α therefore never exceeds 45°). "b" the medium side subtending the angle β, and "c" the hypotenuse.

Given any two of the sides to determine the angles.

Q.1a. Given "a" = 160 and "b" = 231, determine "α".

$$\frac{a}{b} = \text{Tan } \alpha, \text{ } i.e., \text{ } \alpha = \text{Tan}^{-1}\frac{a}{b} = \frac{160}{231}$$

Hence, to determine α (the angle less than 45°), move K to D160 and arrange C231 to K, place K at C10 and read the value of $\frac{a}{b}$ at K on D. ... *Viz.* 0·6925

Bring I50·6925 to K and read the magnitude of "α" on the D scale at C1... ... ... = 34·7° or 34° — 42'

$$\beta = (90 - \alpha) = 55·3° \text{ or } 55° - 18'$$

Q.1b. Given a = 312, c = 555, determine α.

$$\frac{a}{c} = \text{Sin } \alpha \qquad \alpha = \text{Sin}^{-1}\frac{a}{c} = \frac{312}{555}$$

K to D312, C555 to K, K to C10 and read the value of $\frac{a}{c}$ at K on D., *viz.* 0·562. Bring I50·562 to K and read the magnitude of α on the D scale at C1 or C10

$$= 34·2° \text{ or } 34° - 12'$$

Q.1c.   Given b = 46·8, c = 61·5, β required.

Inasmuch as the Inverse Sine scale is from 0 to 1, *i.e.*
for the complete angle range 0° to 90°, β may be
determined as in Exercise 1b. from $\beta = \text{Sin}^{-1}\dfrac{b}{c} = \dfrac{46\cdot8}{61\cdot5}$

$$\beta = 49\cdot55° \text{ or } 49° - 33$$

In each of the previous exercises it will be observed that the
angle magnitude is obtained from the D scale in degrees and
decimals of a degree. For science and engineering computations
this is usually the most desirable form. Such results are readily
converted at sight to angles in degrees and minutes by applying
the factor 60 to the decimal portion of the degree.

e.g., 0·45 of a degree $= (0\cdot45 \times 60)$ or $(4\cdot5 \times 6)$ minutes
$= 27$ minutes.

vice versa,

51 minutes $= \dfrac{51}{60}$ of a degree $= 0\cdot85$ of a degree.

When given any one side and an angle (other than the right angle), to determine the remaining sides of the right angled triangle.

All variations of this case can readily be solved in stages by
such manipulations as those detailed in Examples 2 and 3,
page 7. Alternatively, the following method, based on the
relationship for right-angled triangles that:

$$\frac{a}{a} = \frac{b}{\mathfrak{S}a} = \frac{c}{\mathfrak{S}a}$$

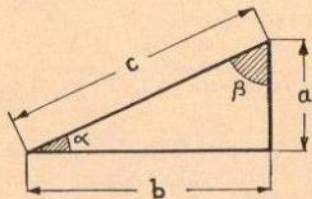and which is expressed diagrammatically by ·
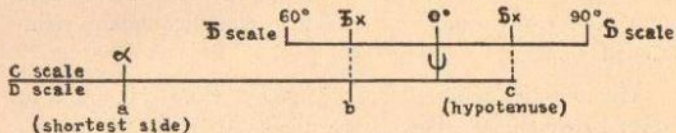
should prove useful inasmuch as the dimensions of both
of the remaining sides are obtainable from a single setting of
the slide ("end switching" excepted, in which case a second
movement is necessary, see S.2b.).

Q.2a. Given c = 533, α = 35° − 18', to determine and a b. [Note, if instead of α, β is given and is greater than 60°, first obtain α from (90 − β) and operate with (the least angle) to ensure being within the Ƨ range]. Place K at D533 and set the slide so that Ƨ35·3° is at K.

Move K to Ƨ35·3 and read "b" on D at K ... 435
Move K to C35·3 and read "a" on D at K ... 308

Q.2b. Given b = 930, α = 38° − 21', a and c required.
K at D930, Ƨ38·35 to K.
K to C38·35 and read "a" on D at K 736
[In order to read the value of "c" on the D scale, it is now desired to position the cursor at Ƨ38·35°. As this is impracticable with the slide so placed, move K to C1 and C10 to K].
K to Ƨ38·35 and read "c" on D at K 1186

Q.2c. Given a = 225, 39° − 36', to obtain b and c.
K at D225, C39·6 to K.
K to Ƨ39·6 and read "b" on D at K 272
K to Ƨ39·6 and read "c" on D at K 353

As evaluations of the types given in exercises 2a, 2b, and 2c so frequently occur in regard to rectangular components of vectors, horizontal distances and vertical heights that correspond to measurements on the incline, etc., it is desirable that the user should memorise the diagrammatic expression of the relationship on which the method is based. This method also provides a ready means of "checking" complete data of right-angled triangles.

The following tables give the "integer" sides a, b, and c, of several right-angled triangles, also the angle "α" (to the nearest minute) which subtends the shortest side "a". These combinations may be used to furnish numerous practice exercises by selecting from any particular set two of the given dimensions and determining the remaining pair by means of the Slide-Rule.

a	b	c	α	a	b	c	α
28	45	53	31° − 53′	87	416	425	11° − 49′
48	55	73	41° − 7′	204	253	325	38° − 53′
39	80	89	25° − 59′	225	272	353	39° − 36′
36	77	85	25° − 3′	93	476	485	11° − 3′
60	91	109	33° − 24′	252	275	373	42° − 30′
85	132	157	32° − 47′	336	377	505	41° − 42′
52	165	173	17° − 29′	185	672	697	15° − 23½
60	221	229	15° − 11′	315	572	653	28° − 51′
308	435	533	35° − 18′	616	663	905	42° − 54′
385	552	673	34° − 54′	348	805	877	23° − 23′
300	589	661	27° − 0′	580	741	941	38° − 3′
432	665	793	33° − 0′	372	925	997	21° − 54′

Oblique-angled Triangles.

Give the lengths of all sides of a triangle, to determine the magnitude of the angle which is opposite a particular side.

Q.3.   Given a = 541, b = 648, c = 423.   To obtain the angle opposite the side "b", *i.e.*, ∠B.

Where S = half of the perimeter of the triangle.

$$S = \frac{a + b + c}{2} = \frac{541 + 648 + 423}{2} = 806$$

11
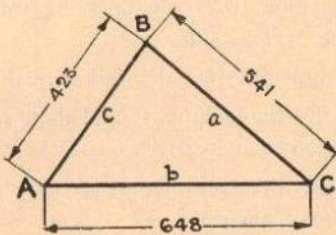
$$\text{Sin}\frac{B}{2} = \sqrt{\frac{(S-a)(S-c)}{ac}}$$

$$= \sqrt{\frac{(806-541)(806-423)}{541 \times 423}} = \sqrt{\frac{265 \times 383}{541 \times 423}}$$

Evaluate $\dfrac{265 \times 383}{541 \times 423}$ on the A and B scales (observing

denominations) and read the root of the result on the
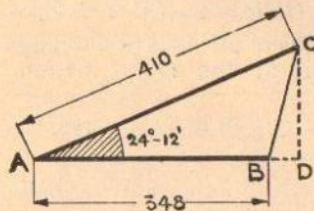
D scale. *Viz.* 0·666

Set K at D666, IS0·666 to K

Read the value of $\dfrac{B}{2}$ on D at C1 ... . 41·75°

or the value of B on D at C2 .. .. 83·5°

Given two sides and the included angle, to determine the remaining side and angles.

Q.4. From the data supplied on the accompanying figure,
determine the length of the side BC also the angles
ACB and CBA.

Imagine CD perpendicular to AB produced if necessary
After operation (i) it will be known whether D lies
within AB or is outside.

(i) Completely solve the right-angled triangle ADC.

(ii) Derive the length BD and then solve the right-angled triangle BDC.

(iii) From the information obtained compute the desired angles.

i.e. (i)   In the △ ACD, AC $=410$, CAD $=24\cdot2°$
Set the slide as explained in Q.2a, and read
AD $=374$, CD $=168$.

(ii)   Then BD $=374-348=26$.
In the △ BDC, BD $=26$, CD $=168$

$$\text{Tan BCD}=\frac{26}{168} \quad i.e., \quad \angle \text{BCD}=\text{Tan}^{-1}\frac{26}{168}$$

Evaluate $\angle$ BCD as Q.1a ...   ... $=$ $8\cdot8°$
         $\angle$CBD $=(90-8\cdot8)$ ... $=$ $81\cdot2°$

$$\frac{\text{CD}}{\text{BC}}=\text{Sin CBD}, \quad \text{BC}=\frac{168}{\text{Sin }81\cdot2°}=\frac{168 \, \text{S} \, 81\cdot2}{81\cdot2}$$

Evaluate BC as Example 3   ... $=$ $170\cdot0$

(iii) $\angle$ ABC $=(180-81\cdot2)$ ...   ... $=$ $98\cdot8°$
      $\angle$ ACB $=(81\cdot2-24\cdot2)$ ...   ... $=$ $57°$

To solve a triangle given one side and two angles.

Q.5.   Given side c $=309$, A $=34\cdot7°$, B $=64°$
Determine sides "a" "b" and "c".
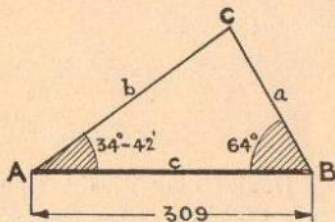
First obtain $\angle$ C from $\angle$ C $=180-(A+B)$
                            $=180-98\cdot7°=81\cdot3°$

$$\frac{a}{\text{Sin A}}=\frac{b}{\text{Sin B}}=\frac{c}{\text{Sin C}}$$

$$a=c\cdot\frac{\text{Sin A}}{\text{Sin C}}=c\cdot\frac{A}{\text{S}_A}\times\frac{\text{S}_C}{C}$$

$$=309\,\frac{34\cdot7}{\text{S}34\cdot7}\times\frac{\text{S}81\cdot3}{81\,3}=178$$

$$b=\frac{\text{Sin B}}{\text{Sin C}}=\frac{B}{\text{S}_B}\times\frac{\text{S}_C}{C}$$
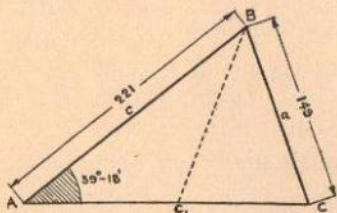
$$=309\,\frac{64}{\text{S}64}\times\frac{\text{S}81\cdot3}{81\cdot3}=281$$

To solve a triangle, given two sides and an angle (not the included angle).

Q.6. Given c = 221, a = 149, A = ∠ 39° − 18′
Determine side "b" also angles B and C.
The accompanying figure is an approximate scale drawing for the given data. It will be noted that there are two triangles that will satisfy the conditions, namely ABC and ABC₁.

Considering the △ ABC.

$$\frac{\text{Sin } C}{c} = \frac{\text{Sin } A}{a} \ , \ \therefore \text{Sin } C = !\frac{c \text{ Sin } A}{a} = \frac{c A}{a \, \mathsf{S} A}$$

$$\angle C = \text{Sin}^{-1}\left(\frac{221 \times 39 \cdot 3}{149 \times \mathsf{S} 39 \cdot 3°}\right)$$

Place K to D221, C149 to K, K to C39·3, 𝖲 39·3 to K, K to C1 (or C10) and note D scale reading. *Viz.* 0·939
𝗜𝗦 0·939 to K and read ∠ C on the D scale at C1 (or C10) ... ... ∠ C = 70°

∠ AC₁B = 180° − 70°	...	...		=	110°
∠ ABC = 180 − (A + C) = 180 − (39·3 + 70)			=	70·7°
∠ ABC₁ = ∠ BCC₁ − ∠ A = 70° − 39·3° ...		=	30·7°

$$\text{Side } AC = \frac{a \text{ Sin } B}{\text{Sin } A} = \frac{a . B . \mathsf{S} A}{\mathsf{S} B . A}$$

$$= \frac{149 \times 70 \cdot 7 \times \mathsf{S} 39 \cdot 3}{\mathsf{S} 70 \cdot 7 \times 39 \cdot 3} \quad \dots \quad \dots \quad = \quad 222$$

$$\text{Side } AC_1 = \frac{149 \times 30 \cdot 7 \times \mathsf{S} 39 \cdot 3}{\mathsf{S} 30 \cdot 7 \times 39 \cdot 3} \quad \dots \quad \dots \quad = \quad 120$$

TACHEOMETRIC SURVEYING
Stadia Computations

On page 9, the complete solution of the right angle triangle is explained in detail when one side and one angle (other than the right angle) are given. The outstanding feature of effecting these computations by means of trigonometrical differential scales is that only a single setting of the slide is necessary.

Inasmuch as the computations of heights and distances from stadia data call for the treatment of two overlapping right angle triangles, it will be seen from the following that a rule which incorporates the differential Sine and Tangent scales provides the best, *in fact the only means*, of effecting the evaluation of D, v and Z (see figure) when G and V° are known, by a *single setting of the slide* (occasional end switching excepted when the differential scales are related to the D scale for maximum accuracy as in the model described).

In the following it is assumed that the tacheometric instrument used is provided with an analatic lens, also that the measurement staff is held in a vertical position.
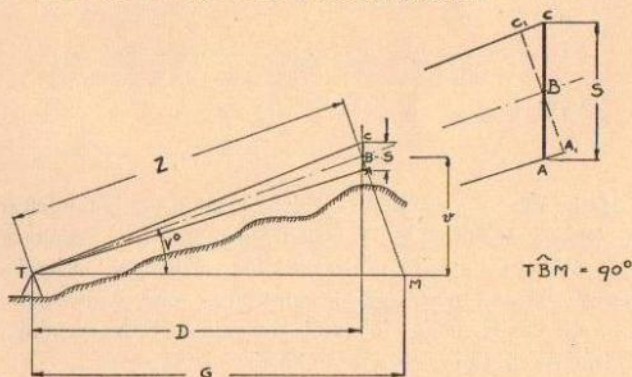
Where V° = the inclination of the telescope.
     S = the (vertical) staff reading = AC.

r = the "corrected" (staff) reading = AC_1
D = the horizontal distance.
v = the vertical distance.
Z = Distance on Collimation Line.
G = Uncorrected distance Z = k.s.
k = Constant for the wires (usually 100).

Q.7.

Determine D, v and Z, when $S = 4\cdot5$ ft. and $V° = 20°$
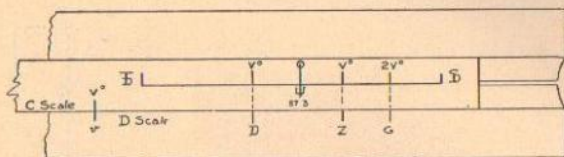and the constant for the stadia lines = 100.

$G = 100 \times 4\cdot5 = 450$, $V° = 20°$, $2V° = 40°$

Set the slide so that the $40°$ mark on the S scale aligns with 450 on the D scale.

Then in alignment with :

$20°$ on the S scale read the value of $Z = 422\cdot9$ on the D scale				
$20°$	do. S scale	do.	$D = 397\cdot4$	do.
20	do. C scale	do.	$u = 144\cdot6$	do.

That is evaluate in accordance with the following diagram

In the case of a value of G which occurs towards the left of the D scale such that S $2V°$ when aligned with G results in $SV°$, $SV°$ or $CV°$ being to the left of D.1., then end switching must be effected by placing the cursor at C_{10} and then bringing C_1 to the cursor. This will enable the values of the remaining projections on the D scale to be made.

The mathematical proof which substantiates the correctness of the foregoing method follows :

From the figure and remembering that
$$\text{Sin } V° \text{ Cos } V° = \frac{\text{Sin } 2V°}{2}$$

we get :
$$v = G \text{ Sin } V° \text{ Cos } V° = \text{Tan } V° \text{ G.Cos}^2 V° = D \text{ Tan } V°$$
$$= Z \text{ Sin } V° = \frac{G. \text{ Sin } 2V°}{2}$$

Then
$$\frac{V°}{v} = \frac{V°}{G. \text{ Sin } V° \text{ Cos } V°} = \frac{V°}{\text{Tan } V°.G.\text{Cos}^2 V°} = \frac{V°}{D.\text{Tan } V°}$$
$$= \frac{V°}{Z \text{ Sin } V°} = \frac{2V°}{G. \text{ Sin } 2V°}$$

$$\frac{V°}{v} = \frac{\dfrac{V°}{\text{Tan } V°}}{D} = \frac{\dfrac{V°}{\text{Sin } V°}}{Z} = \frac{\dfrac{2V°}{\text{Sin } 2V°}}{G}$$

i.e.
$$\frac{V°}{v} = \frac{\text{s}V°}{D} = \frac{\text{s}V°}{Z} = \frac{\text{s}2V°}{G} \text{ or } \frac{\text{s}2V°}{\text{k.s.}}$$

The slide rule interpretation of this form is given in the previous figure.

When the angle $V°$ is small (say less than $3°$), then within the accuracy limits of measuring "S".

(i) $S \simeq r$ (ii) $Z \simeq 100r$

(iii) $D \simeq Z$ (iv) $v \simeq \dfrac{100rV°}{57\cdot3}$

Q. 8   Given $V° = 1° - 14'$ (*i.e.* less than $3°$), and $r = 6\cdot17$
To determine D and "v"
Express $1° - 14'$ as a degree and decimal $= 1\cdot233°$
$$D \simeq 100 \times 6\cdot17 = 617$$
To obtain "v" evaluate $\dfrac{D.V°}{57\cdot3}$ $\dfrac{617 \times 1\cdot233}{57\cdot3}$

Set K at D617, U (at $57\cdot3$) to K, K to $1\cdot233$
Read "v" at K on D ...   . $13\cdot28$ ft.

Gauge Marks U, m, and s.

These are angle conversion constants, engraved on the slide adjacent to the C scale (to which they relate), respectively positioned according to the following values and when used as divisors serve for the primary purpose stated:

$$U = \frac{180}{\pi} = 57\ 2958 = \text{Degrees to Radians.}$$

$$m = \frac{180 \times 60}{\pi} = 3437\ 75 = \text{Minutes to Radians.}$$

$$s = \frac{180 \times 60 \times 60}{\pi} = 206265 \cdot 0 = \text{Seconds to Radians.}$$

That is, where α° is an angle expressed in Degrees.

α' the angle expressed in Minutes.

α'' do. do. Seconds.

ψ do. do. Radians.

then :

$$\frac{\alpha^\circ}{U} \text{ or } \frac{\alpha'}{m} \text{ or } \frac{\alpha''}{s} = \psi$$

The form of the latter relationship, if memorised, will serve to direct the use of these constants.

When an angle is small (say less than 3 degrees)

$$\text{Sine } \alpha \fallingdotseq \psi \fallingdotseq \text{Tan.} \alpha$$

approximations which give the constants U, m and s an added importance as will be seen from the evaluation effected in Exercise 9.

Q. 9 Express the following angles in radians: (a) $26^\circ\ 42'$, (b) 17 8 minutes, (c) 36 5 secs.

(a) $26^\circ 42'$ treat as $26 \cdot 7^\circ$

Radian measure $\psi_a = \dfrac{26 \cdot 7}{57 \cdot 2958} = \dfrac{26 \cdot 7}{U}$

Set K at D267 and bring U to K, read ψ_a on the D scale at (C1 or) C10   ...   ...   0·466 radians.

(b) 17·8 minutes $= \psi_b$ radians,

$$\psi_b = \frac{17·8}{3437·75} = \frac{17·8}{m}$$

Set K at 17·8 on D and bring m to k, read ψ_b on the D scale at (C1 or) C10 ...   ... 0·00518 radians.

(c) 36·5 secs. $= \psi_c$ radians,

$$\psi_c = \frac{36·5·}{206265} = \frac{36·5}{s}$$

K to 36 5 on D and s to K, read ψ_c on D at C1 (or C10)   ...   .   0·00177 radians

Q.10 Determine the Sine and Tangent values of $1°-17'$ (less than $3°$).

Treat $1°-17'$ as 77 minutes.

Then $\psi_{1°-17'}$, Sin $1°-17'$, Tan $1°-17' \simeq \dfrac{77}{3437·75} = \dfrac{77}{m}$

K to 77 on D and m to K, read $\psi_{1°-17'}$, Sine and Tangent for $1°-17'$ on D at C1 (or C10) ... 0·0224

Differential Scales
Theoretical Considerations

The Differential Scales, previously referred to, are selected Logarithmic Scales of $\frac{x}{f(x)}$ which can readily be manipulated as divisor correctives in conjunction with a Logarithmic Scale of "x" in such a manner as to cancel out the "x" and leave the "f(x)" operative

$$i.e., \quad \frac{x}{\dfrac{x}{f(x)}} \quad = f(x)$$

The Differential Scale mode of applying a "Function" to the Slide Rule is advantageous where, for a range of "x" under consideration, the logarithmic scale length that is required for $\frac{x}{f(x)}$ is relatively short in comparison to that which would be required for "f(x)."

Considerations of a particular "function" will elucidate these statements

Where "a" is an angle in degrees
and "$f(a)$" = Sin a.

For the range $a = 0°$ to $a = 90°$, Sin a ranges from 0 to 1. A complete logarithmic scale of Sin x would be infinitely long, a "rule length" being required for each "cycle of number significance" such as 0·001 to 0·01, 0·01 to 0·1, and 0·1 to 1, the respective Sines for the angle ranges $3' - 26''$ to $34' - 23''$, $34' - 23''$ to $5° - 44' - 21''$ and $5° - 44' - 21''$ to 90°.

For the corresponding angles, the values of $\frac{a}{f(a)}$ range from 57:3 to 90 or 1 to 1·571 which is only a fraction of a "cycle of number significance" and in consequence only requires a fraction of a "rule length" to completely express it.

The design and nature of the Sine Differential Scale, also the necessary manipulative mode will be readily understood on

inspection of the Angle-Sine Logarithmic Scale Graph (Fig. 2) and perusal of the following remarks.

The lines of the Direct Differential Sine Scale, dimensioned by their corresponding angles, are situated on the slide at such distances as gh, g_1h_1, g_2h_2, from the index C.10. Orthodox Sine Scales comprise lines at such distances as mh, m_1h_1, m_2h_2, m_3h_3, etc., measured from the index C.1.

It will be observed that the variations in length of gh, g_1h_1. etc., are relatively small for the angle range 90° to 10° and that below 5° such lengths become approximately constant and ultimately furnish a distance for the zero position (at C57 3) of the Sine Differential Scale; whilst such lengths as m h, m_1h_1, m_2h_2, etc., undergo considerable change (in cycles).
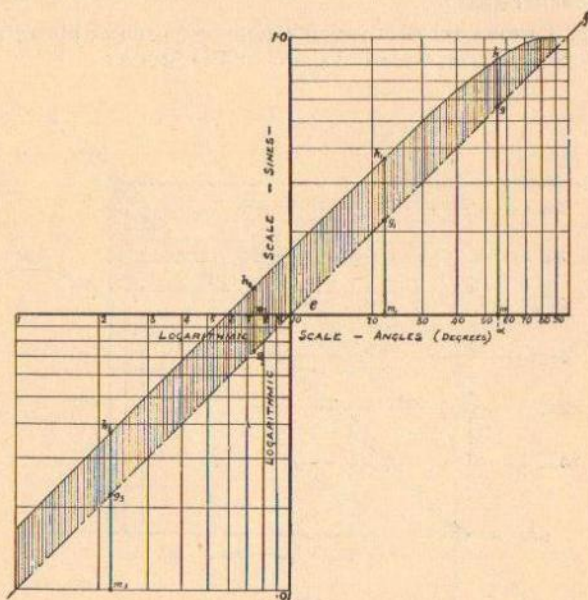

Fig. 2

Referring to Fig. 2 :

$$g\,h = m\,h - m\,g$$
$$h\,g = m\,g - m\,h = e\,m - m\,h$$
$$= \text{Mantissa of Log } a - \text{Mantissa of Log sin } a$$
$$= \text{Log} \frac{a}{\text{Sin } a} \pm \frac{\text{Characteristic}}{\text{correction.}}$$

$$= \text{Log} \frac{a}{10 \sin a} = \text{Log} \frac{a}{\sin a} - 2$$

In such cases as this, where the mantissa of Log Sin a is greater than that of Log a, *i.e.*, when the graph is above "e f" giving "h g" a negative value, the line of the Sine Differential Scale is situated at a distance "g h" to the left of C.10; but for "functions" whose graphs fall below "e f" the vertical deviations from "e f" are measured to the right from C.1. The Inverse Sine and Tangent Differential Scales furnish examples of the latter group.

Fig. 3 shows the Derivation Graphs for each of the Differential Scales incorporated in the C and D Models.
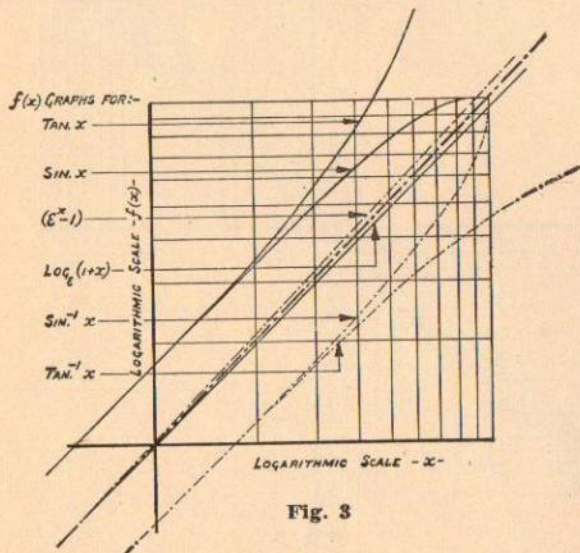

Fig. 3